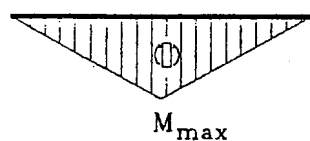
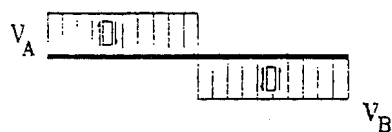
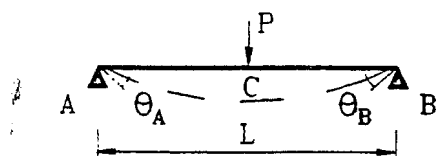


**UNIVERSIDAD PONTIFICIA COMILLAS
ESCUELA TÉCNICA SUPERIOR DE INGENIERÍA (ICAI)**

**INGENIEROS INDUSTRIALES
3^{er} CURSO**

ELASTICIDAD Y RESISTENCIA DE MATERIALES

Formulario de vigas



Giros:

$$\theta_A = \theta_B = \frac{1}{16EI} PL^2$$

Flecha:

$$f_C = \frac{1}{48EI} PL^3$$

Fuerza cortante:

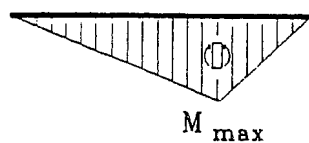
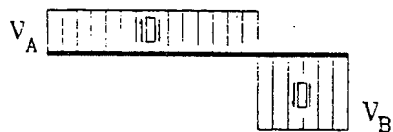
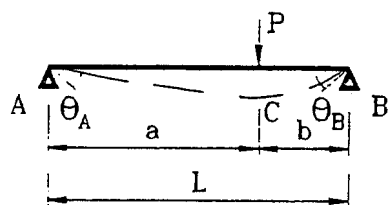
$$V_A = V_B = \frac{P}{2}$$

Momento flector:

$$M_{\max} = \frac{PL}{4}$$

Ecuación de la elástica:

$$y_{AC} = \frac{1}{16EI} PL^2 x \left(1 - \frac{4x^2}{3L^2} \right)$$



Giros:

$$\theta_A = \frac{1}{6EI} \frac{Pab(L+b)}{L} \quad \theta_B = \frac{1}{6EI} \frac{Pab(L+a)}{L}$$

$$\theta_C = \frac{1}{3EI} \frac{Pab(a-b)}{L} \quad \Delta + \theta_C$$

Flechas:

$$f_C = \frac{1}{6EI} PLab \left(1 - \frac{a^2}{L^2} - \frac{b^2}{L^2} \right)$$

$$f_{\max} = \frac{1}{9\sqrt{3}EI} \frac{Pb(L^2 - b^2)^{3/2}}{L} \quad \text{para } x = \sqrt{\frac{L^2 - b^2}{3}} \quad (a > b)$$

Fuerza cortante:

$$V_A = \frac{Pb}{L} \quad V_B = \frac{Pa}{L}$$

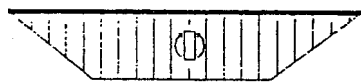
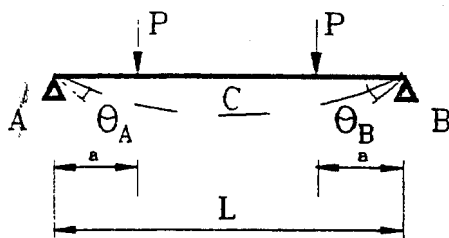
Momento flector:

$$M_{\max} = \frac{Pab}{L}$$

Ecuación de la elástica:

$$y_{AC} = \frac{1}{6EI} PLbx \left[1 - \frac{b^2}{L^2} - \frac{x^2}{L^2} \right]$$

$$y_{CB} = \frac{1}{6EI} PLa(L-x) \left[1 - \frac{a^2}{L^2} - \left(\frac{L-x}{L} \right)^2 \right]$$



$M_{\max}$

Giros:

$$\theta_A = \theta_B = \frac{1}{2EI} P a (L - a)$$

Flecha:

$$f_c = \frac{1}{EI} P a \left(\frac{L^2}{8} - \frac{a^2}{6} \right)$$

Fuerza cortante:

$$V_A = V_B = P$$

Momento flector:

$$M_{\max} = P a$$

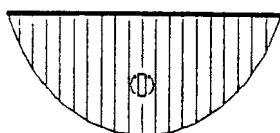
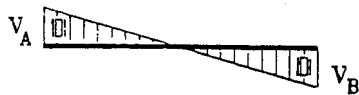
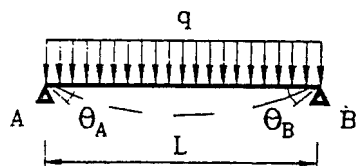
Ecuación de la elástica:

para $x < a$

$$y = \frac{1}{EI} P \left(-\frac{x^3}{6} + \frac{aL}{2}x - \frac{a^2}{2}x \right)$$

para $a < x < (L - a)$

$$y = \frac{1}{EI} P \left(-\frac{a^2}{2}x + \frac{aL}{2}x - \frac{a^3}{6} \right)$$



$M_{\max}$

Giros:

$$\theta_A = \theta_B = \frac{1}{24EI} q L^3$$

Flecha:

$$f_{\max} = \frac{5}{384EI} q L^4$$

Fuerza cortante:

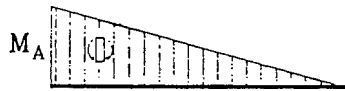
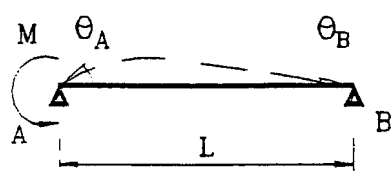
$$V_A = V_B = \frac{qL}{2}$$

Momento flector:

$$M_{\max} = \frac{qL^2}{8}$$

Ecuación de la elástica:

$$y = \frac{1}{24EI} q x (x^3 - 2Lx^2 + L^3)$$



Giros:

$$\theta_A = \frac{1}{3EI} ML \quad \theta_B = \frac{1}{6EI} ML$$

Flecha:

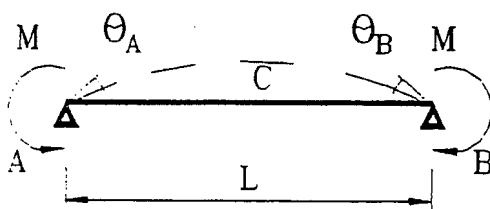
$$f_{\max} = \frac{ML^2}{9\sqrt{3}EI} \quad \text{para } x = L \left(1 - \frac{1}{\sqrt{3}} \right)$$

Fuerza cortante:

$$V = \frac{M}{L}$$

Ecuación de la elástica:

$$y = -\frac{1}{6EI} ML(L-x) \left[1 - \left(\frac{L-x}{L} \right)^2 \right]$$



Giros:

$$\theta_A = \theta_B = \frac{1}{2EI} ML$$

Flecha:

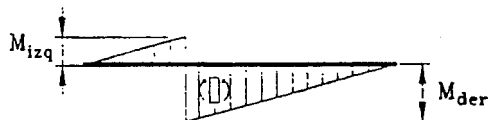
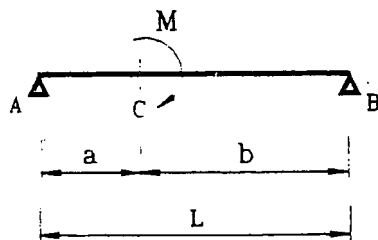
$$f_C = \frac{1}{8EI} ML^2$$

Fuerza cortante:

$$V = 0$$

Ecuación de la elástica:

$$y = -\frac{1}{2EI} M(L-x)x$$



Giros:

$$\theta_A = \frac{1}{6EI} ML \left(3 \frac{b^2}{L^2} - 1 \right) \quad \theta_B = \frac{1}{6EI} ML \left(3 \frac{a^2}{L^2} - 1 \right)$$

$$\theta_C = \frac{1}{3EI} \frac{M}{L^2} (a^3 + b^3) \quad \Delta = \theta$$

Flecha:

$$f_C = \frac{1}{3EI} \frac{Ma b}{L} (b - a) \quad \downarrow + f_c$$

Fuerza cortante:

$$V = \frac{M}{L}$$

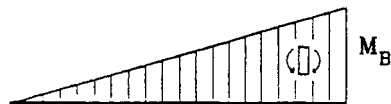
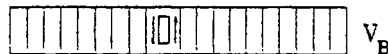
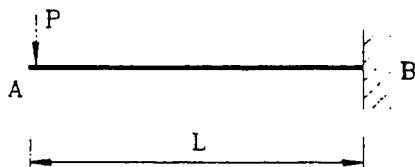
Momento flector:

$$M_{izq} = \frac{M}{L} a \quad M_{der} = \frac{M}{L} b$$

Ecuación de la elástica:

$$y_{AC} = -\frac{1}{6EI} ML x \left(1 - 3 \frac{b^2}{L^2} - \frac{x^2}{L^2} \right)$$

$$y_{BC} = \frac{1}{6EI} ML (L - x) \left[1 - 3 \frac{a^2}{L^2} - \left(\frac{L - x}{L} \right)^2 \right]$$



Giro:

$$\theta_A = \frac{1}{2EI} PL^2$$

Flecha:

$$f_A = \frac{1}{3EI} PL^3$$

Fuerza cortante:

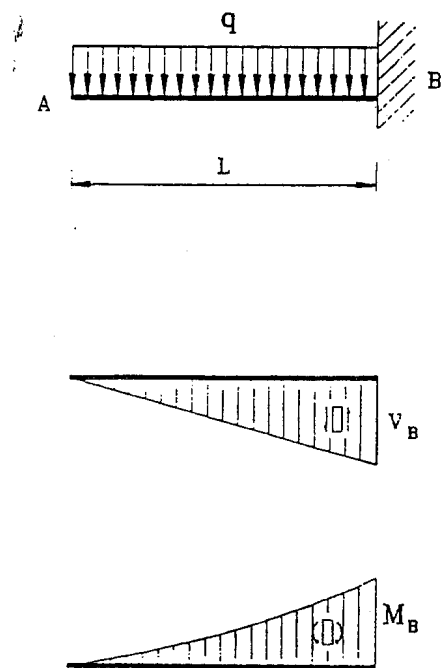
$$V = P$$

Momento flector:

$$M_B = PL$$

Ecuación de la elástica:

$$y = \frac{1}{6EI} P (L - x)^2 (2L + x)$$



Giro:

$$\theta_A = \frac{1}{6EI} q L^3$$

Flecha:

$$f_A = \frac{1}{8EI} q L^4$$

Fuerza Cortante:

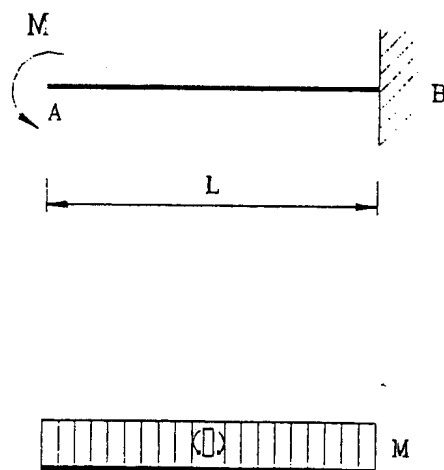
$$V_B = q L$$

Momento Flector:

$$M_B = \frac{1}{2} q L^2$$

Ecuación de la elástica:

$$y = \frac{1}{24EI} q (L - x)^2 (3L^2 + 2Lx + x^2)$$



Giro:

$$\theta_A = \frac{1}{EI} M L$$

Flecha:

$$f_A = \frac{1}{2EI} M L^2$$

Fuerza cortante:

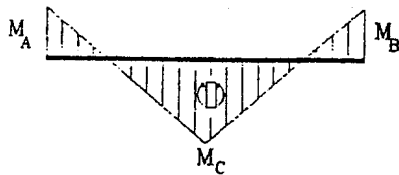
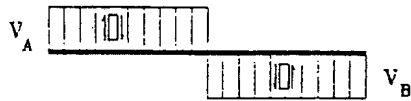
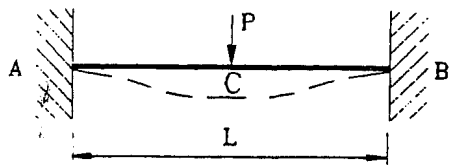
$$V = 0$$

Momento flector:

$$M_f = M$$

Ecuación de la elástica:

$$y = \frac{1}{2EI} M (x^2 - 2Lx + L^2)$$



Flecha:

$$f_c = \frac{1}{192EI} PL^3$$

Fuerza cortante:

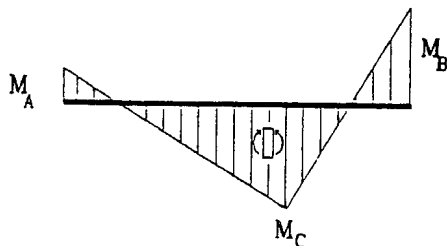
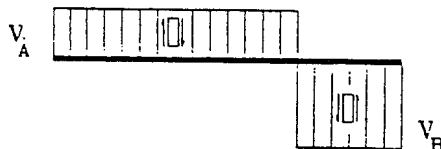
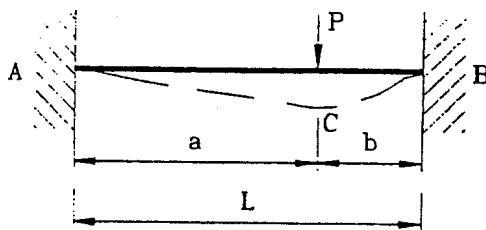
$$V_A = \frac{P}{2} \quad V_B = \frac{P}{2}$$

Momento flector:

$$M_A = M_B = M_C = \frac{PL}{8}$$

Ecuación de la elástica:

$$y_{AC} = \frac{1}{48EI} PLx^2 \left(3 - 4 \frac{x}{L} \right)$$



Giro:

$$\theta_c = \frac{1}{2EI} \frac{Pa^2b^2}{L^2} \left(\frac{2a}{L} - 1 \right) \quad \Delta_c + \theta_c$$

Flechas:

$$f_c = \frac{1}{3EI} \frac{Pa^3b^3}{L^3}$$

$$f_{max} = \frac{2}{3EI} \frac{Pb^2a^3}{(L+2a)^2} \quad \text{para} \quad x = \frac{2aL}{L+2a}$$

Fuerza cortante:

$$V_A = \frac{Pb^2}{L^3} (L+2a) \quad V_B = \frac{Pa^2}{L^3} (L+2b)$$

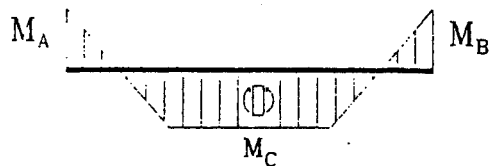
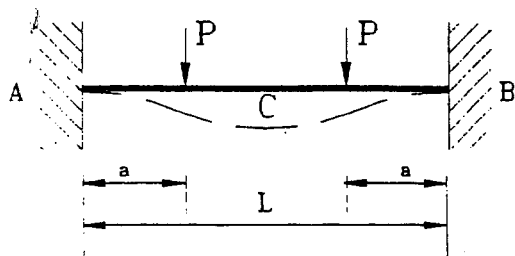
Momento flector:

$$M_A = \frac{Pa^2b^2}{L^2} \quad M_B = \frac{Pa^2b^2}{L^2} \quad M_C = \frac{2Pa^2b^2}{L^3}$$

Ecuación de la elástica:

$$y_{AC} = \frac{1}{6EI} Pb^2 \left(3a - x - \frac{2ax}{L} \right) \frac{x^2}{L^2}$$

$$y_{BC} = \frac{1}{6EI} Pa^2 \left[3b - (L-x) - 2b \frac{L-x}{L} \right] \frac{(L-x)^2}{L^2}$$



Flecha:

$$f_C = \frac{1}{EI} P \left[-\frac{a^4}{L} - \frac{3}{8} a^2 L + \frac{7}{6} a^3 \right]$$

Fuerza cortante:

$$V_A = V_B = P$$

Momento flector:

$$M_A = M_B = P a \frac{(L-a)}{L} \quad M_C = \frac{P a^2}{L}$$

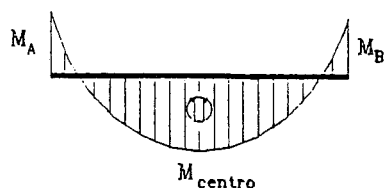
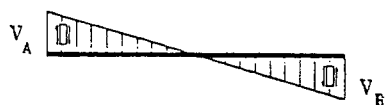
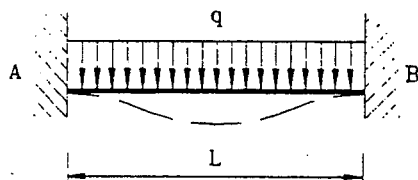
Ecuación de la elástica:

para $x < a$

$$y = \frac{1}{EI} P \left(\frac{x^3}{6} + a \frac{(L-a)}{2L} x^2 \right)$$

para $a < x < L-a$

$$y = \frac{1}{EI} P \left[\frac{a^2}{2L} x^2 - \left(\frac{a^2}{2} - a^2 \frac{(L-a)}{L} + \frac{a^3}{L} \right) x + \frac{a^4}{L} + \frac{a^3}{6} \right]$$



Flecha:

$$f_{\max} = \frac{1}{384EI} q L^4$$

Fuerza cortante:

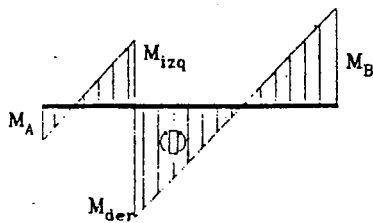
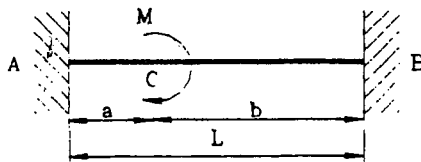
$$V_A = V_B = \frac{qL}{2}$$

Momento flector:

$$M_A = M_B = \frac{1}{12} q L^2 \quad M_{\text{centro}} = \frac{1}{24} q L^2$$

Ecuación de la elástica:

$$y = \frac{1}{24EI} q L^4 \left(\frac{x}{L} - \frac{x^2}{L^2} \right)^2$$



Giro:

$$\theta_C = \frac{1}{EI} \frac{Mab}{L^3} (a^2 + b^2 - ab) \quad \Delta + \theta_C$$

Flecha:

$$f_C = \frac{1}{2EI} \frac{Ma^2b^2}{L^3} (a - b)$$

Fuerza cortante:

$$V = \frac{6M}{L^3} ab$$

Momento flector:

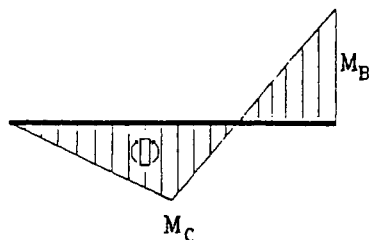
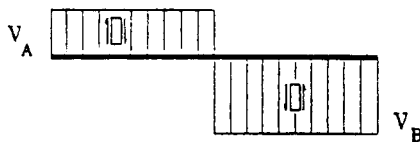
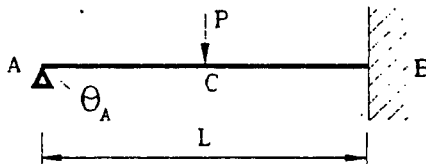
$$M_A = \frac{Mb}{L} \left(2 - 3 \frac{b}{L} \right) \quad M_B = \frac{Ma}{L} \left(2 - 3 \frac{a}{L} \right)$$

$$M_{\max} = M_A - \frac{6M}{L^3} a^2b \quad M_{\text{der}} = M_A + \frac{M}{L^3} (L^3 - 6a^2b)$$

Ecuación de la elástica:

$$y_{AC} = \frac{1}{2EI} \frac{Mb}{L} x^2 \left(2a \frac{L-x}{L^2} - \frac{b}{L} \right)$$

$$y_{BC} = \frac{1}{2EI} \frac{Ma}{L} (L-x)^2 \left(2 \frac{bx}{L^2} - \frac{a}{L} \right)$$



Giros:

$$\theta_A = \frac{1}{32EI} PL^2 \quad \theta_C = \frac{1}{128EI} PL^2 \quad \Delta + \theta_C$$

Flechas:

$$f_C = \frac{7}{768EI} PL^3 \quad f_{\max} = \frac{1}{48\sqrt{5}EI} PL^3 \quad \text{para } x = \frac{L}{\sqrt{5}}$$

Fuerza cortante:

$$V_A = \frac{5}{16} P \quad V_B = \frac{11}{16} P$$

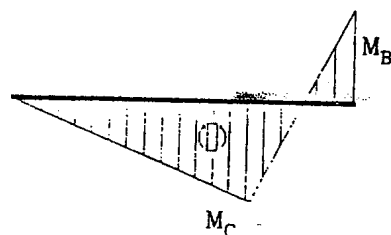
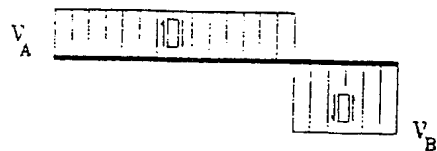
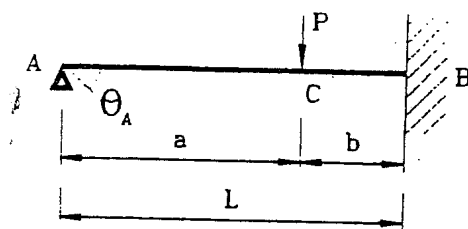
Momento flector:

$$M_B = \frac{3}{16} PL \quad M_C = \frac{5}{32} PL$$

Ecuación de la elástica:

$$y_{AC} = \frac{1}{96EI} PL^2 x \left(3 - 5 \frac{x^2}{L^2} \right)$$

$$y_{CB} = \frac{1}{96EI} PL(L-x)^2 \left(9 - 11 \frac{L-x}{L} \right)$$



Giros:

$$\theta_A = \frac{1}{4EI} \frac{Pa(L-a)^2}{L}$$

$$\theta_C = \frac{1}{4EI} \frac{Pa(L-a)^2}{L^3} (L^2 - 2aL - a^2) \quad \Delta - \theta_C$$

Flechas:

$$f_C = \frac{1}{12EI} \frac{Pa^2b^2}{L^3} (3L^2 - 2aL - a^2)$$

$$f_{\max} = \frac{1}{6EI} Pb^2a \sqrt{\frac{a}{2L+a}} \quad \text{para } x = L \sqrt{\frac{a}{2L+a}}$$

Fuerza cortante:

$$V_A = \frac{Pb^2}{2L^3} (3L - b) \quad V_B = \frac{Pa}{2L^3} (3L^2 - a^2)$$

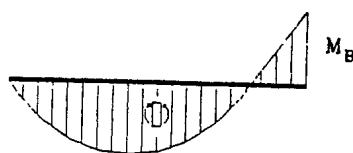
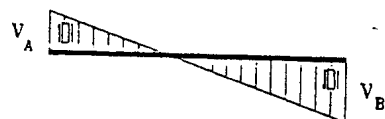
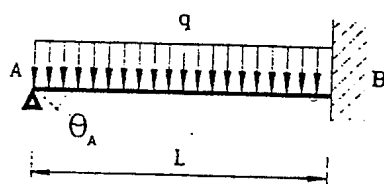
Momento flector:

$$M_B = \frac{Pa}{2L^2} (L^2 - a^2) \quad M_C = \frac{Pa}{2L^3} b^2 (3a + 2b)$$

Ecuación de la elástica:

$$y_{AC} = \frac{1}{12EI} \frac{Pb^2x}{L^3} [3aL^2 - x^2(2L+a)]$$

$$y_{CB} = \frac{1}{12EI} Pa(L-x)^2 \left[3 \left(1 - \frac{a^2}{L^2} \right) - \left(3 - \frac{a^2}{L^2} \right) \frac{L-x}{L} \right]$$



Giro:

$$\theta_A = \frac{1}{48EI} qL^3$$

Flecha:

$$f_{\max} = \frac{1}{185EI} qL^4 \quad \text{para } x = \frac{1+\sqrt{33}}{16} L$$

Fuerza cortante:

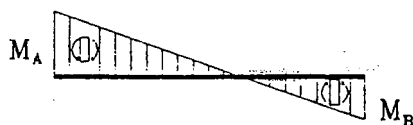
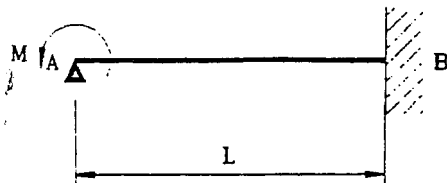
$$V_A = \frac{3}{8} qL \quad V_B = \frac{5}{8} qL$$

Momento flector:

$$M_B = \frac{qL^2}{8} \quad M_{\max} = \frac{9}{128} qL^2 \quad \text{para } x = \frac{3}{8} L$$

Ecuación de la elástica:

$$y = \frac{1}{48EI} qx(L+2x)(L-x)^2$$



Giro:

$$\theta_A = \frac{1}{4EI} ML$$

Flecha:

$$f_{\max} = \frac{1}{27EI} ML^2 \quad \text{para } x = \frac{L}{3}$$

Fuerza cortante:

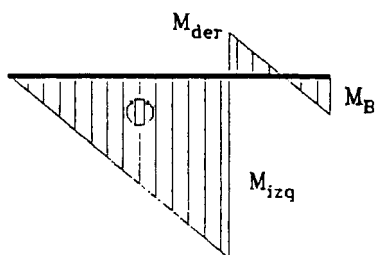
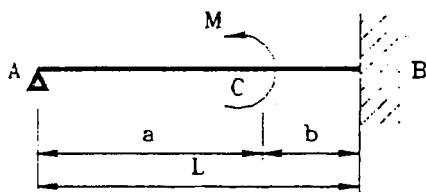
$$V_A = V_B = \frac{3}{2} \frac{M}{L}$$

Momento flector:

$$M_A = M \quad M_B = \frac{M}{2}$$

Ecuación de la elástica:

$$y = \frac{1}{EI} ML \left[\frac{2L}{27} \left(1 - \frac{3}{2} \frac{x}{L} \right)^3 + \frac{1}{12} x - \frac{2}{27} L \right]$$



Giros:

$$\theta_A = \frac{1}{4EI} \frac{M(L-a)(L-3a)}{L} \quad \Delta + \theta_A$$

$$\theta_C = \frac{1}{4EI} Mb \left[4 - 3 \frac{b}{L} \left(1 + \frac{a}{L} \right)^2 \right] \quad \Delta + \theta_C$$

Flecha:

$$f_C = \frac{1}{4EI} \frac{Mab^2}{L^3} (2a^2 - b^2) \quad \Delta + f_C$$

Fuerza cortante:

$$V_A = \frac{3}{2} \frac{M}{L^3} (L^2 - a^2)$$

Momento flector:

$$M_{\text{der}} = V_A a - M \quad M_{\text{izq}} = V_A a$$

$$M_B = \frac{M}{2L^2} (L^2 - 3a^2)$$

Ecuación de la elástica:

$$y_{AC} = \frac{1}{4EI} \frac{Mbx}{L^3} [-4L^3 - (x^2 - 3L^2)(a+L)]$$

$$y_{BC} = \frac{1}{4EI} \frac{M}{L^3} (L - x)^2 [2a^2L - x(L^2 - a^2)]$$